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NURSERY EXAMPLES OF FRACTIONS.

BY MRS. BOOLE.

AT the risk of seeming monotonous, I must preface my present paper by repeating what I have already said in this journal :—The intellectual work of learning arithmetic belongs to the schoolroom ; it should be done as work, and by methods devised by competent teachers. No amateur can improve upon the plans already in use to give children facility in the manipulation of arithmetical expressions. There is nothing in arithmetic that need prove an undue tax on the powers of any average child, were not the real difficulties complicated by being entangled with others which are imaginative, or one might almost say moral. If, just when a child has to face in class a new intellectual problem, it is also confronted with one of different order, which puts a strain on some other part of the nature, neither difficulty can engross the whole attention ; and the result is the setting up of an irregular double action of the mind, an uneven sort of cross-strain, which is unfavourable both to proper comprehension of the subject in hand, and to the general mental health. Those who have charge of the child in the nursery, and later on in vacations, can obviate beforehand the evils of this imaginative interference with intellectual action, *not* by attempting to teach arithmetic, but by confronting him with questions which can be solved by his natural common sense without any special mental effort, and yet will accustom his imagination to situations similar in kind to those which will necessitate mental effort later on.

The chief imaginative difficulties which entangle themselves with that department of arithmetic known as "fractions" are of three kinds :—First, an apparent arbitrariness in the answers admitted and in the amount of accuracy required. For instance, one seventh of one pound five and sixpence may be variously expressed as "three shillings and sevenpence and five-sevenths of a penny" ; "three and sevenpence halfpenny and six-sevenths of a farthing" ; or "three and sevenpence halfpenny and six over." There is not the least intellectual difficulty in understanding that these three answers are arithmetically equivalent, provided that the child is free to attend to the arithmetical demonstration ; but many children feel vaguely (what is quite true, though teachers sometimes forget it) that the three answers correspond to different ways of treating the financial fact, different attitudes towards it, different conceptions of its nature. Nothing in mathematical philosophy is more beautiful than the way in which a set of equivalent arithmetical expressions reflect various possible conceptions of the same action ; every child should be free to soak in this marvel, in silence and at his own pace. But he cannot do that while some one is explaining "division of money" to him ; if he is confronted with the spiritual revelation and the intellectual problem at the same moment, he will miss the enjoyment of the one, and his facility in mastering the other will suffer also ; if he has previously been accustomed to perceive the relation between *fractional* and *residual* treatment of remainders, he will have nothing to attend to when the need for choice between them occurs in class, except the demonstration of formal equivalence in the particular case.

The second difficulty is that caused by the distinction between "proper," "improper," and "mixed" fractions. The third is due to the apparent anomaly that multiplying a quantity by a (proper) fraction diminishes that quantity, and dividing it by a fraction increases it.

We have to prepare the child's imagination for all these metaphysical complications. The following experiences, or others like them, would meet the requirements of the case :—

A cake is to be divided equally between two children ; each will have half a cake. Next day the cake is rather larger ; the amount available for each will be really larger ; but the process of dividing and the name of the share (one half) will be exactly the same as before.

Another day four cakes are to be shared, weighing about as much as one of the former cakes. The process is now different ; we have to count instead of cutting. The expression has also changed ; we say "two cakes," not "half-a-cake." But the amount is the same as before.

Four apples are to be divided between the children ; each has two apples. Five apples are to be divided ; then each receives two, and the remaining apple must be cut. (Here we have a combination of both modes of division.) Suppose that the five apples are cooking apples ; and are to be shared between two "little cooks" (who will probably not be the babies who are being initiated into the mysteries of fractions, but older children whom the babies are eagerly watching). The dish in process of preparation may be one that requires the fruit to be cut in halves; each cook will then have five half-apples to deal with; but the cutting has not altered the amount of apple material. Another day the apples may be cut into quarters; and another day into thirds.

Two little cooks have been given leave at a farm to take home all the eggs they can find; they have found five eggs. Minnie shall make a pudding with two to-day, and Beckey shall use two to-morrow in a cake. But what shall we do with the fifth egg? We might break it to-day and divide it in two, and each child would then have two eggs and a half. But it would be inconvenient and messy; we had better leave one egg "over." It can be given to someone from the two children in common, or it can be left over till more eggs come in and can then form a part of a new distribution.

Finally, a single lily bulb may be given to two children, with the suggestion that it is to be shared between them. They may thus be led to see that a live thing cannot be divided; the only thing that can be shared is the pleasure of watching it grow—a possession which is not diminished by being enjoyed by more than one person participating in it.

One point to bring out in this series of object lessons is that such a question as "What is one-half of so-and-so?" may involve either one of several questions. It may mean "What *weight* (or bulk) of cake is half the weight of five cakes?" Answer:—"Twice the weight of one cake plus half the weight of one cake." Or, "What *number* of eggs can each of two persons use out of five eggs?" Answer:—"Two eggs; and there will be one over for some other purpose." Or, "What advantage will come to each of two persons, when each receives half of what was a living healthy bulb weighing five ounces?" Answer:—"Each will receive two and a half ounces of useless, dying vegetable material." The mother who has furnished her children's mind-chambers with clear images of the various kinds of halving, will have done more to facilitate their future study of fractions than she could have done by any amount of premature explanation of numerical processes.

Before a child begins to work fractions in school, it should be practically familiar with the fact that the larger the denominator the smaller the fraction. It is easy to call the attention of a very tiny child to the principle that one-sixth is smaller than one-fifth; he ought to recognize that the word "sixth" in "one-sixth" represents the number of parts into which something is to be divided, and has no relation to the quantity to be divided If something is divided into six parts and then shared between three children, they will see that "sixth," in the expression "two-sixths," represents an act of division, whereas "two" represents the number of sixths that falls to the share of each child; also that such share is essentially equal—in weight, bulk, value—to the one piece which each would have received had the original object been divided into only three. It may be also made clear that if the object be originally cut into seven pieces, there will be no possibility of equally distributing those pieces among three persons without further cutting; the seventh piece must be either left "over," or else cut into three; it can be shewn, without reasoning or arguing, that two-sevenths plus one-third of a seventh comes to the same value as one-third of the whole.

The meaning of such expressions as "one-half of a third," "one-fifth of one-fourth," etc., may be further illustrated by dividing a party of children into groups of three or four, each under a captain, and sharing something between the little captains, to be divided by them among their respective groups. In all this work explanations should be avoided; the aim should be to familiarize the children with actual processes, and to link those processes in their minds with the terms which they will afterwards use in connection with arithmetical operations. But in order that this may be done in the right sequence and with good effect, it is, of course, necessary that the person who directs the sequence of processes should mentally connect them with clear conceptions of arithmetical operations.

We have still to deal with the third source of difficulty which I have mentioned in connection with fractions: the fact that the operation called multiplication has a diminishing effect when the multiplier is a proper fraction. Nothing in mathematics is more important to get clear than the inversion of the effect of many operations when the line is crossed which separates from each other the two conceptions :—*More-than-one* and *less-than-one*. It would simplify this whole subject if children were accustomed occasionally to use the expression "half-a-child" instead of "one hand" in little problems connected with "handfuls," *e.g.*: "If each child can get six walnuts out of a bag by dipping with his hands, how many will be got out by four children? three children? two children? one child? half-a-child?" Children easily get into the habit of treating grotesque questions of this kind as fun, see through them and answer them correctly, and thus gain the habit of leaping the metaphysical fence in play, before it occurs to them to think of it as an obstacle.

I think if teachers realized how many of the difficulties of arithmetic are due to our faulty terminology, they would try to devise a better one. The words "multiplication" and "division," as names for arithmetical operations, are unsound and misleading. If the operation called "multiplying" be performed by means of one number *larger than unity* (the so-called multiplier) on another (the multiplicand), the effect is indeed to multiply the latter; but if the "multiplier" be either unity, zero, a proper fraction, a geometric line, or a transcendental expression, its effect is quite different from multiplying. It is a pity that we have not some name for the operation, suited to characterize it truly throughout and cause no misconceptions. The operation has been well defined as "doing to the second of two expressions what, if done to unity, would have generated the first." This definition applies equally to all forms of the operation miscalled multiplying (whether numerical, fractional, negative, geometric, or transcendental), and I wish some name could be agreed on which would not tend to confuse the passage from one branch of mathematics to another.

Something may be done, however, with our present terminology, bad as it is, to diminish the difficulty of passing from units to fractions. Attention might be called to the fact that when a little captain receives half a cake, and then shares it between three children, he diminishes the size of the pieces but does really multiply the number of parts into which the cake was originally cut.

A set of play-questions which I have found useful is of this kind :—"Suppose there are three ponies in a field, how

many heads, eyes, tails, legs, feet, hoofs, hands? And how many bits of mischief do you think would be done in that field in the day? Suppose a boy came into the field; then how many heads, eyes, tails, . . . bits of mischief? Suppose two boys came in; how many heads, eyes, tails, etc.? Suppose one of the boys had a monkey on his shoulder; how many heads, eyes, . . . bits of mischief?" This kind of exercise gets children accustomed to realize which elements in a statement are relevant to the special question asked (*e.g.*: the addition of a boy to the group of ponies alters the number of heads, but not the number of hoofs). Many children, clever at actual calculations, fail miserably in algebra examinations (and in many more important crises of life), for lack of agility in detecting what is, and what is not, relevant to the special point under consideration.

It is also necessary that children should learn early to distinguish between questions which can be exactly answered (the number of heads or hoofs) and questions which only appeal to one's imagination and one's general sense of probability (the amount of mischief that two boys and a monkey would be likely to do). Direct instruction on the limits of the knowable should of course not be attempted in childhood; but by the two kinds of questions being mixed together in play, the children will gain the habit of discriminating.

The questions about the ponies and monkey do not properly belong to the domain of fractions, but I have introduced the topic here, because what I have said about it may help to make clear a few remarks I have to offer upon the psychology of such a lesson as the mock-proposition to fractionalize a lily-bulb. Such a proposition should be made simply as a bit of fun; in the same spirit in which the mother asks whether she shall throw baby over the wall into the neighbour's garden. It should take its place as a stock nursery joke, which strikes the imagination of the elder children by its obvious absurdity, and which the little ones laugh at (and therefore remember) because they see the elders laughing. What we need to produce, in relation to such a matter, is not intellectual conviction, but a vivid and abiding mental picture, an "unconscious constant factor" in the mind, a crystallizing thread round which future wisdom may gather and organize itself. What ails mathematicians (the whole history of political economy is there to prove that something ails them very badly) is not that any of them doubt the existence of a boundary line beyond which mathematical formulæ no longer apply,—all of them, when the question is mooted, acknowledge that such a boundary exists,—but that their knowledge of it lies too far back in their minds and does not crop up readily enough into consciousness. A politician might be saved from many an error, by a picture cropping up at the right moment to the surface of his thoughts: his mother proposing to distribute the future flowers by cutting the bulb into bits, or asking him to find out by the multiplication-table how much more mischief two boys can do together than either would be likely to invent alone.

I am well aware that many people, while acknowledging that appliers of mathematics to life sorely need to be more aware than most of them are of the exact limits of their science, suppose that lack of clear consciousness of those limits does not affect the actual business of learning to use the arithmetical formulæ. I am convined that this is a grave mistake. Practical contact with the lines of demarcation traced by Nature should precede, not follow, the acquisition of knowledge of humanly-devised formulæ intended to facilitate the classification of facts; experience has forced me to recognise that this is as true in arithmetic as in any other department of study. It may be said of arithmetical formulæ as of all other implements of human devising:—The best preparation for understanding their use is to be well trained to perceive the precise limits of their usefulness. Some teachers object to all such preparation, on the ground that it makes class-work dull by forestalling the main source of its attractiveness. But no preparation can possibly discount the legitimate influence of the teacher who knows his business; what we can discount is that non-legitimate influence which is won by suddenly throwing a flash of light upon an imagination hitherto darkened. The more one sees of the effect of such irregular illumination on neurotic young people, the more one desires to arm them against it, by enabling them to perceive truth gradually, as it presents itself before them, in the steady light of a sound mathematical philosophy.

OBEDIENCE.

III. WILLING OBEDIENCE.

By Henry Beveridge.

We will now turn our attention to the moral side of obedience. Moral or willing obedience is differentiated from the obedience of compulsion by the presence of motives implying freedom of will, and expressed by such modes of speech as "I ought"; "I will, because it is right"; "I must, because it is in accordance with the law of my life." It is sanctioned no longer by any external authority, but solely by the inner self-conscious judgment of the performer of the act; outward compulsion is replaced by the inner sense of duty, or, higher still, by a compelling sense of fitness which has become one alike with the judgment of reason and the free impulse of emotion. At the back of the judgments, "I ought," "I will," "I must," there is the consciousness of freedom, and at the back of the act which follows is an inward commanding force,—the force which we call will. The act thus initiated is called moral or immoral, in the narrower sense, according as it coincides or disagrees with certain fixed standards or ideals—on higher or lower planes it may be—of what is good or what is right. But is this willing obedience, this compelling sense of fitness, purely the fruit of an inward spontaneous activity, or may it arise by stimulation from without? It is clear at any rate that it cannot be forced into existence by any outside compulsion. Compulsion, as we have seen, produces compulsory obedience; it may also produce habit through many repetitions, but mere automatic habit clearly contains no element of duty: at the back of all dutiful activity there must be free, active and conscious will. How then does the sense of duty arise? The answer is surrounded with difficulty. I would venture to say, however, that the sense of duty cannot otherwise be aroused save through the perception of a solidarity of relation among the separate parts of a social unity. When such a perception has been formed, there necessarily arises further the conception of a law or natural order whose bonds, more or less formulated, are conceived as binding the separate parts into a related whole. Just as it is the necessity